Seminar Dedicated to the

80th Anniversary of Gennady Zinovjev

Deformation of two-particle spectra due to interaction in the final state

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Outline of the talk

1. Introduction

2. Relativistic effects in FSI

3. Distortion of the Coulomb FSI due to high multiplicities

Common works:

- D.V. Anchishkin, M.I. Gorenstein, G.M. Zinovjev,
  Cumulative Effect and the Model of Nuclear Fireballs,
  Phys. Lett. B, v. 108, No. 1, p. 47 (1982).

- D. Anchishkin and G. Zinovjev,
  Two-pion correlation behavior in a small relative momentum region, Phys. Rev. C, v.51, No. 5, p. R2306 (1995).

- D.V. Anchishkin, W.A. Zajc, G.M. Zinovjev,
  Coulomb corrections in two particle correlations for the processes of high multiplicity,
  Ukr.J.Phys. 41, pp. 363-369 (1996); arXiv: hep-ph/9512279.

- D.V. Anchishkin, W.A. Zajc, G.M. Zinovjev,
  The Influence of High Multiplicities at RHIC on the Gamov Factor, arXiv: nucl-th/9904061 [nucl-th].

- D.V. Anchishkin, W.A. Zajc, G.M. Zinovjev,
  High Multiplicities Influence on the Pion-Pion Final State Interactions in Relativistic Heavy Ion Collisions,
  Ukr.J.Phys. 46, No.12, pp. 1-9 (2001).

Two-particle correlations

$$C(\mathbf{q},\mathbf{K}) = \frac{P_2(\mathbf{p}_a,\mathbf{p}_b)}{P_1(\mathbf{p}_a)\,P_1(\mathbf{p}_b)}\ ,$$

where

$$P_2(\mathbf{p}_a,\mathbf{p}_b) = E_a E_b \frac{d^6 N}{d^3p_a d^3p_b}, \quad P_1(\mathbf{p}) = E\,\frac{d^3N}{d^3p} \qquad E = \sqrt{m^2+\mathbf{p}^2}\,,$$

We are looking for two-particle probability to registrate particles with certain momenta $\mathbf{p}_a$ and $\mathbf{p}_b$:

$$P_2(\mathbf{p}_a,\mathbf{p}_b) = ?$$

And then,

$$C(\mathbf{q},\mathbf{K}) = ?$$

$$\mathbf{K} = \frac{1}{2}(\mathbf{p}_a + \mathbf{p}_b)\,, \qquad \mathbf{q} = \mathbf{p}_a - \mathbf{p}_b$$

Some approximations: $\mathbf{v}_a \approx \mathbf{v}\,, \quad \mathbf{v}_b \approx \mathbf{v}$.
In the center-of-mass system of the pair $\mathbf{v} = 0$,

$$P_1(\mathbf{p}) = \int d^4x\, S\,(x, p)$$

$$P_2(\mathbf{q}) = \int d^4x_a\, d^4x_b\, S\,(x_a, p_a)\; S\,(x_b, p_b)\; \left|\phi_{\mathbf{q}/2}(\mathbf{x}_a - \mathbf{x}_b)\right|^2 \pm$$

$$\pm \int d^4x_a\, d^4x_b\, S\,(x_a, K)\; S\,(x_b, K)\; \phi^*_{\mathbf{q}/2}(\mathbf{x}_b - \mathbf{x}_a)\, \phi_{\mathbf{q}/2}(\mathbf{x}_a - \mathbf{x}_b)\,,$$

where $\phi_{\mathbf{q}/2}(\mathbf{x}_a - \mathbf{x}_b)$ is solution of the Schrödinger equation for a relative evolution of two particles under the Coulomb interaction.

Physical meaning: *Two single-particle probabilities to find particles in the time-space points x_a and x_b with certain momenta $\mathbf{p}_a$ and $\mathbf{p}_b$, which are expressed by $S(x,p)$, is weighted by the probability $\left|\phi_{\mathbf{q}/2}(\mathbf{x}_a - \mathbf{x}_b)\right|^2$ to find these particles with relative distance $\mathbf{x}_a - \mathbf{x}_b$ and relative momentum $\mathbf{q}$.*

The correlation function reads:

$$C(\mathbf{q}) = \frac{P_2(\mathbf{q})}{\int d^4x_a \, S\left(x_a, p_a\right) \int d^4x_b \, S\left(x_b, p_b\right)} \, ,$$

where 4-vectors $p_a = (\mathbf{q}^2/4m, \, \mathbf{q}/2)$ and $p_b = (\mathbf{q}^2/4m, \, -\mathbf{q}/2)$.

In the non-interacting limit (only the symmetry of the two-particle wave function is taken into account): $\phi_{\mathbf{q}/2}(\mathbf{x}) \to \exp\left(i\mathbf{q} \cdot \mathbf{x}/2\right)$

$$C(\mathbf{q}, \mathbf{K}) = 1 \pm \frac{\left| \int d^4x \, e^{i\mathbf{q}\cdot\mathbf{x}} S\left(x, K\right) \right|^2}{\int d^4x_a \, S\left(x_a, p_a\right) \int d^4x_b \, S\left(x_b, p_b\right)} \, .$$

Coulomb final state interaction

The model source function:

$$S(x,p) \;\propto\; \exp\left[-\omega(\mathbf{p})/T_{\mathsf{f}} - t^2/2\tau^2 - \mathbf{r}^2/2R_0^2\right]$$

In the pair c.m.s ($\mathbf{K}=0$):

$$P_2(\mathbf{q}) \;\propto e^{-2K^0/T_{\mathsf{f}}} \int d^3r\, e^{-r^2/4R_0^2} \left[\left|\phi_{\mathbf{q}/2}(\mathbf{r})\right|^2 \pm \phi^*_{\mathbf{q}/2}(-\mathbf{r})\,\phi_{\mathbf{q}/2}(\mathbf{r})\right]$$

The single-particle probability reduces to the pure Boltzmann exponent:

$$P_1(\mathbf{k}) \;\propto e^{-\omega(\mathbf{k})/T_{\mathsf{f}}}$$

The Gamow Factor and relativistic effects

$$C(\mathbf{p}_a, \mathbf{p}_b) \;=\; G(\mathbf{p}_a, \mathbf{p}_b)\, C_{\mathsf{model}}(\mathbf{p}_a, \mathbf{p}_b).$$

Correlation function without FSI ($\mathbf{K} = 0$):

$$C(\mathbf{q}) = 1 + e^{-\mathbf{q}^2 R_0^2}$$

Correlation function, corrected by the Gamov factor $G(|\mathbf{q}|)$:

$$C(\mathbf{q}) \;=\; G(|\mathbf{q}|) \left(1 \,+\, e^{-\mathbf{q}^2 R_0^2}\right) \,,$$

where

$$G(|\mathbf{q}|) \;=\; \left|\phi_{\mathbf{q}/2}(\mathbf{r} = 0)\right|^2 .$$

Pure Coulomb:

$$G(|\mathbf{q}|) = \frac{2\pi\eta}{e^{2\pi\eta} - 1}$$

with

$$\eta \;=\; \frac{\alpha\, m_\pi}{|\mathbf{q}|}$$

Penetration through the Coulomb barrier : $\eta_0 \equiv \dfrac{e^2 m_\pi}{Q} = \eta_{pen}(r_1 = 0, r_2)$

where $V_{\mathrm{Coul}}(r_2) = E^0_{\mathrm{kin}}$ and

$$\eta_{\mathrm{pen}}(r_1, r_2) = \frac{1}{\pi} \int_{r_1}^{r_2} dr\, q(r)\,, \quad \text{or} \quad \eta_{\mathrm{pen}}(r_1, r_2) = \frac{1}{\pi} \left[I(r_1) - I(r_2) \right] .$$

In the nonrelativistic case the penetration integral:

$$I_{\mathrm{nr}}(r) = rq(r) - \eta_0 \arcsin\left(1 - 2\frac{E^0_{\mathrm{kin}}}{V_{\mathrm{Coul}}(r)}\right), \quad q(r) = \sqrt{2m\left(V_{\mathrm{Coul}}(r) - E^0_{\mathrm{kin}}\right)},$$

and in the relativistic case:

$$\left[(-\boldsymbol{\nabla}^2 + m^2)^{1/2} + V_{\mathrm{Coul}}(r)\right] \psi(\boldsymbol{r}) = (m + E^0_{\mathrm{kin}})\psi(\boldsymbol{r})$$

$$\begin{aligned} I_{\mathrm{rel}}(r) = rq_{rel}(r) \quad &- \quad \gamma\eta_0 \arcsin\left(\gamma - 2\frac{E^0_{\mathrm{kin}}}{V_{\mathrm{Coul}}(r)} - \frac{\left(E^0_{\mathrm{kin}}\right)^2}{m_\pi V_{\mathrm{Coul}}(r)}\right) - \\ &- \quad \alpha_{\mathrm{eff}} \arcsin\left(\gamma - \frac{V_{\mathrm{Coul}}(r)}{m_\pi}\right) \end{aligned}$$

where $q_{\mathrm{rel}}(r) = \sqrt{\left(2m + E^0_{\mathrm{kin}} - V_{\mathrm{Coul}}(r)\right)\left(V_{\mathrm{Coul}}(r) - E^0_{\mathrm{kin}}\right)}$ and
$\gamma = E/m_\pi$ and $\alpha_{\mathrm{eff}} = e^2/2$. Then, $\eta_{\mathrm{pen}}(r_1, r_2) = \frac{1}{\pi}\left[I_{\mathrm{rel}}(r_1) - I_{\mathrm{rel}}(r_2)\right]$.

The Coulomb plus strong two-particle interaction

$$\left[(-\boldsymbol{\nabla}^2 + m^2)^{1/2} + V_{\text{eff}}\right]\psi(\boldsymbol{r}) \;=\; (m + E^0_{\text{kin}})\psi(\boldsymbol{r})\,,$$

it is the version of the Bethe-Salpeter equation for spinless particles.

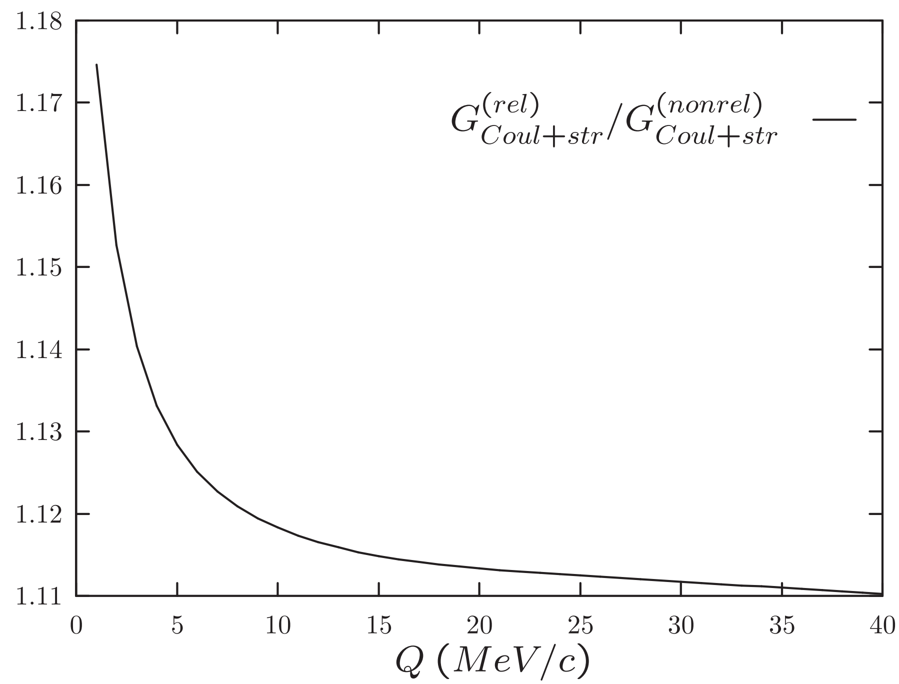
Potential energy:

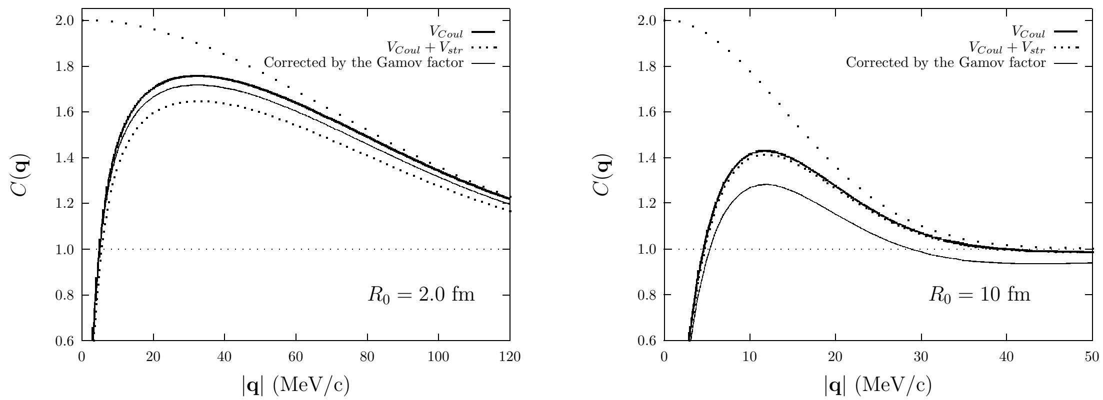
$$V_{\text{Coul}}(r) \;=\; \frac{\alpha}{r}\,, \qquad V_{\text{eff}}(r) \;=\; V_{\text{Coul}}(r) + V_{\text{str}}(r)\,,$$

where

$$V_{\text{str}}(r) = V_0 \frac{e^{-m_\rho r}}{m_\rho r}\,, \qquad V_0 = 2.6 \text{ GeV}, \quad m_\rho = 770 \text{ MeV}\,.$$

Potential of the strong repulsion (S. Pratt et al., PRC **42**, 2646 (1990)) was chosen to match the behavior of the pion-pion scattering phase shifts. In any case the use of this strong potential can be considered as a model of the short-range repulsion which is possessed by pions.


1.18
1.17
1.16
1.15
1.14
1.13
1.12
1.11
0
5
10
15
20
25
30
35
40
$Q\ (MeV/c)$
$G^{(rel)}_{Coul+str}/G^{(nonrel)}_{Coul+str}$



It is seen that the finite size of the emission source softens the manifestation of the FSI and the 'Gamov factor' tends to overestimate the FSI effects for the source of big size ($R_0 \geq 4$ fm).

Final state interactions at high secondary multiplicities

Two-particle potential in dense environment

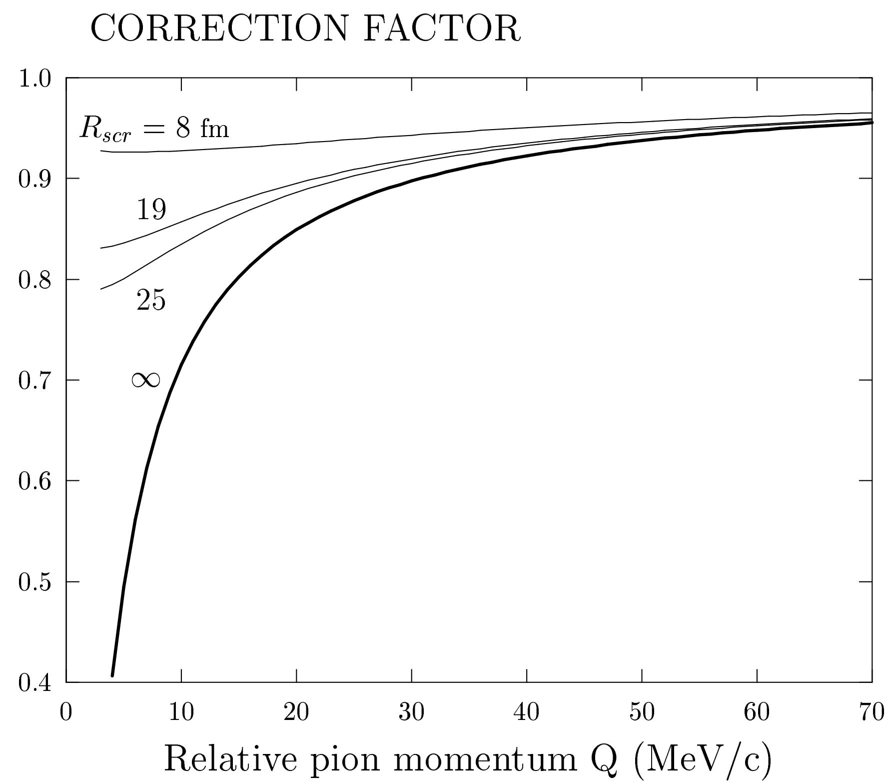
$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\phi(t,\mathbf{r}) \;=\; 4\pi e\left(n^{(+)} - n^{(-)}\right)$$

$$n^{(\pm)} = n^{(0)}\,\exp\left(\mp\frac{e\phi}{T_{\mathsf{f}}}\right), \quad e\phi \ll T_{\mathsf{f}} \quad \Rightarrow \quad n^{(\pm)} = n^{(0)}\,\left(1 \mp \frac{e\phi}{T_{\mathsf{f}}}\right)$$

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\phi(t,\mathbf{r}) \;=\; -\,\frac{8\pi\alpha}{3\,T_{\mathsf{f}}}\,n_\pi\,\phi(t,\mathbf{r})$$

I. Solution of the Schrödinger equation in case of a constant density of the environment

$$U_{\pi\pi}(r) \;=\; \alpha\,\frac{e^{-r/R_{scr}}}{r}, \quad \frac{1}{R_{scr}} \;=\; \sqrt{\frac{8\pi}{3}\alpha}\cdot\sqrt{\frac{n_\pi}{T_f}} \quad \Rightarrow \quad G_{\mathsf{cor}}(Q) \;=\mid \psi(\mathbf{r}=0)\mid^2$$



1) RHIC (LHC): $N_\pi = 8000$, $T_f = 180$ MeV, $R_f = 7.1$ fm, $R_{scr} = 7.9$ fm,
2) SPS-1: $T_f = 187\ MeV$, $\tau_f = R_L \approx 6.0$ fm, $R_T = 6$ fm, $\Delta y = 3$, $dN/dy = 40$, $\Rightarrow$ $R_{scr} = 19.3$ fm,
3) SPS-2: $N_\pi = 800$, $T_f = 190$ MeV, $R_f = 7$ fm, $\Rightarrow$ $R_{scr} = 25$ fm.

Investigation of the post-freeze-out phase density

In order to take into account post-freeze-out expansion of the pion system let us consider a pion phase-space distribution

$$\frac{\partial f(x,p)}{\partial x^0} + \mathbf{v}\cdot\nabla f(x,p) = 0\,, \qquad \mathbf{v} = \frac{\mathbf{p}}{E(\mathbf{p})}\,, \qquad E(\mathbf{p}) = \sqrt{m_\pi^2 + \mathbf{p}^2}$$

$$\text{with} \quad \lim_{t\to\infty} f(t, \mathbf{r}=0; \mathbf{p}) = 0\,, \quad f_0(\mathbf{R},\mathbf{p}) = n_0(\mathbf{R})\, g_0(\mathbf{p})\,,$$

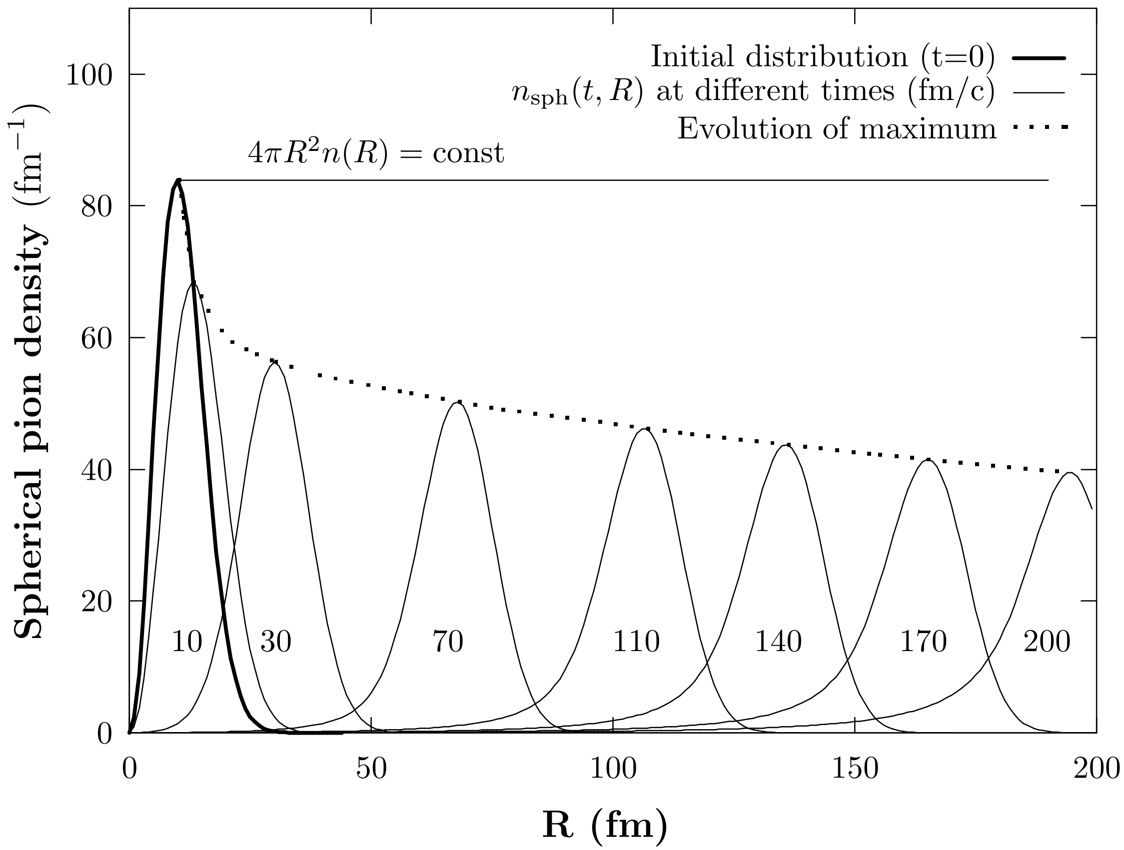
where R is the distance from the fireball center. The spatial distribution of the particles at time t:

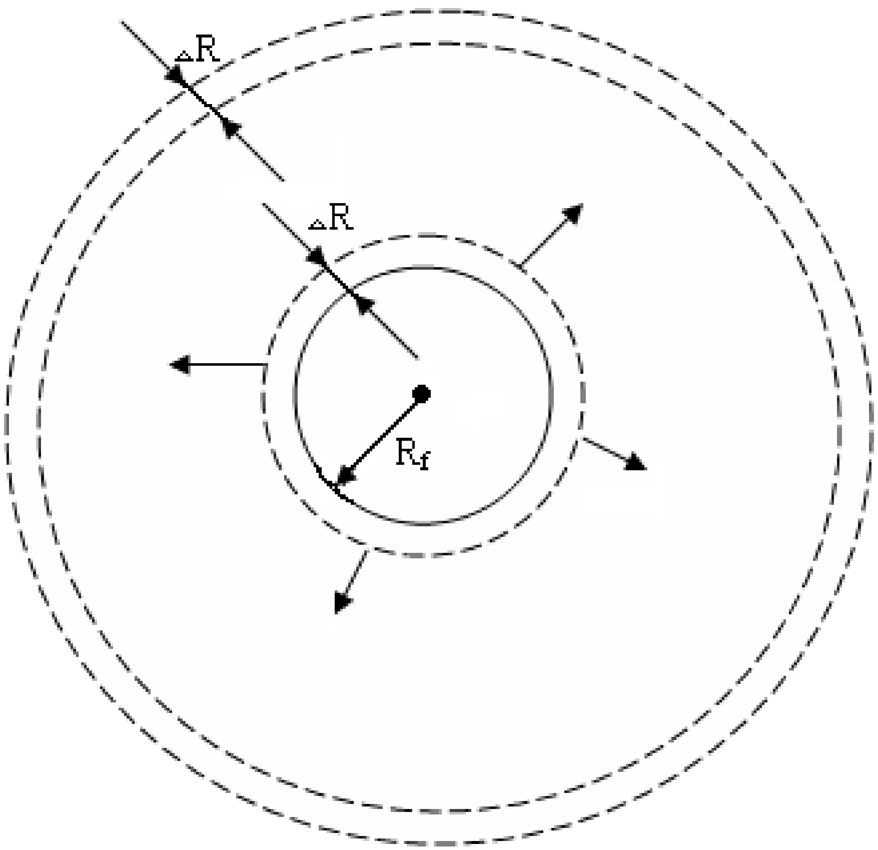
$$n(t,\mathbf{R}) = \int \frac{d^3p}{(2\pi)^3}\, n_0\Big(\mathbf{R} - \frac{\mathbf{p}}{E(\mathbf{p})}\, t\Big)\, g_0(\mathbf{p})$$

The spherical density: $\mathbf{n}_{\mathsf{sph}}(\mathbf{t},\mathbf{R}) = 4\pi\mathbf{R}^2\mathbf{n}(\mathbf{t},\mathbf{R})$,
where $\int_0^\infty dR\, n_{\mathsf{sph}}(t,R) = N_\pi$

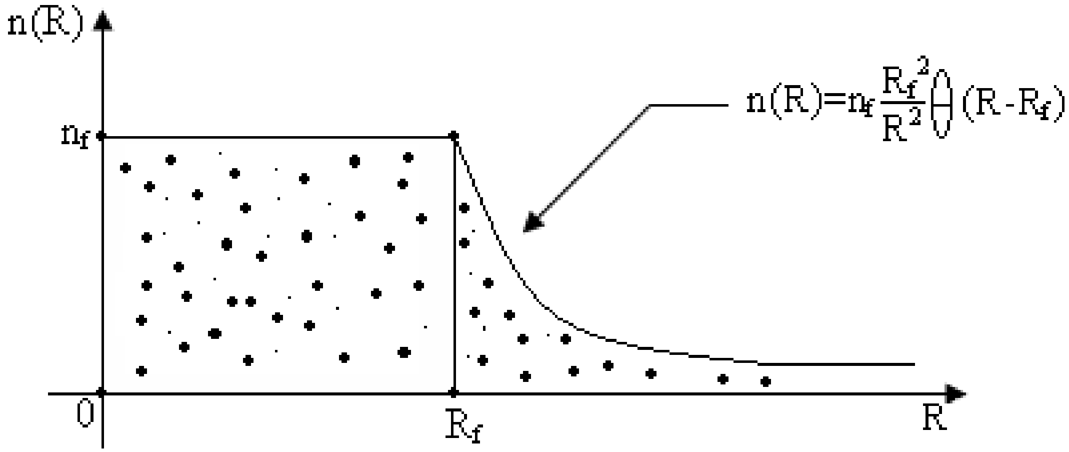
Evolution of 1D spherical density

The spatial distribution at times $t = 10, 30, 70$, *etc.* fm/c




ΔR
ΔR
R_f

II. Modeling the spatial and time dependence of the post freeze-out pion density

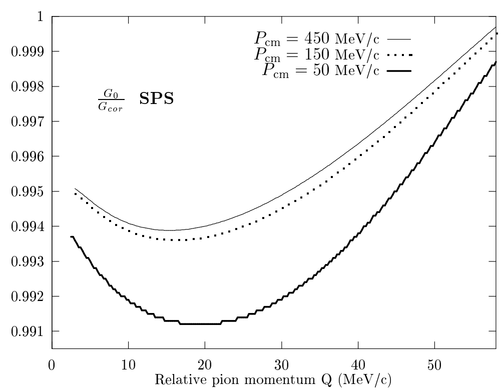


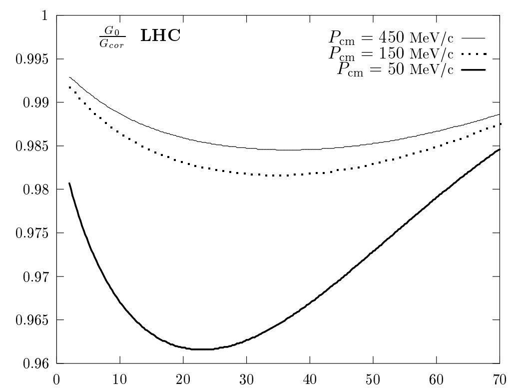
In the spherically expanding system the density of the environment depends on time t and distance from the fireball R.

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) \phi(t, \mathbf{r}) = -\frac{8\pi\alpha}{3\,T_{\rm f}}\, n_\pi(R)\, \phi(t, \mathbf{r})$$

$$R \approx R_{\mathsf{f}} + v_{\mathsf{cm}} \cdot t\,, \qquad r \approx v_{\mathsf{rel}} \cdot t \quad \Rightarrow \quad R = R_{\mathsf{f}} + r\,\frac{v_{\mathsf{cm}}}{v_{\mathsf{rel}}}$$

$$\left(1 - v_{\mathsf{rel}}^2\right)\frac{d^2\phi(r)}{dr^2} + \frac{2}{r}\frac{d\phi(r)}{dr} - \frac{c^2(q)}{(r+\bar{r})^2}\,\phi(r) = 0$$





1) $N_\pi = 1000$, $T_{\mathsf{f}} = 190$ MeV, $R_{\mathsf{f}} = 7$ fm
2) $N_\pi = 8000$, $T_{\mathsf{f}} = 190$ MeV, $R_{\mathsf{f}} = 7$ fm

High secondary multiplicities

Conclusion:

Due to the rapid decrease in the density of the secondary particle medium, the distortion of the Gamow factor, which is taken as an indicator of the Coulomb final state interactions, is almost insignificant, even if the density of the secondary particles is overestimated.

Вітаю ще раз

Геннадія Михайловича з ювілеєм

Дякую за увагу

Thank You for Attention