

**37 років захоплюючих наукових досліджень з
Г. М. Зінов'євим**

Kyryll Alekseevich Bugaev

Outline

- 1. Laboratory of HEDPhysics and BITP: an image in eyes of provincial university student in 1984**
- 2. Development of exactly solvable models of cluster type: from toy model of deconfinement to fundamental findings**
- 3. Negative surface tension, problems with defining critical exponents and non-Fisher universality classes**
- 4. Finite Width of QG bags and new picture of confinement**

Kyiv, April 26, 2021

A lucky chance of mine

In Sept. 1984 Oleg Borisenko, Kyrill Bugaev, **Valentin Shelud'ko and Vladimir Shkira** came to BITP for professional training

were the students of the Dniepropetrovsk National University (DNU)-
A provincial university in Ukraine:

**Rektor: acad. Vladimir Ivanovich Mossakovsky a good friend of
acad. Ostap Stepanovich Parasyuk (since Ph D studentship)**

G.M. Zinovjev met us and briefly presented the results of Laboratory

**I worked with Marina Korkina (prof) and came to P. I Fomin
To establish a common work,**

But P. I Fomin went to a one week business trip...

**Hence G.M. Zinovjev suggested to me not to waste time and «to try to understand
another nonlinear and gauge theory known as lattice formulation of QCD»**

In a week, when P. I Fomin returned, I decided to work on lattice QCD



Three personal reasons to work with G.M. Zinovjev

1. **Fantastic working atmosphere in Laboratory of HEDPhysics:**
From making great physics to cleaning up the BITP territory!
2. **A chance to discuss modern physics with foreign researchers**
3. **I felt like at home, since G. M. Zinovjev, Yu. M. Sinykov and O. P. Pavlenko Graduated from my Alma Mater DNU**



Teatime in 1986:
From left to right:
G.M. Zinovjev, Larry McLerran,
S.I. Lipskih, Yu. M. Sinykov,
myself, A.P. Kobushkin

Gas of Bags: Exactly Solvable Statistical Model of Cluster Type

- * 1965 Hagedorn suggested an exponentially growing mass spectrum for heavy hadrons. The model led to the idea of limiting temperature for hadrons.
- * 1974 MIT Bag model is proposed. It treats hadrons as QG bags.
A.Chodos et. al., Phys. Rev. D 9, (1974) 3471.
- * 1975 Cabbibo and Parisi conjectured that limiting temperature evidences for the new physics above T_H . **The relevant d.o.f. are quarks and gluons.**
- * 1981 Kapusta showed that MIT Bags have the Hagedorn mass spectrum. The Gas of Bags model is suggested. It unifies the three previous ideas.
Hence, heavy hadrons = QGP bags. PRD 23 (1981) 2444.
- * **1981 Gas of Bags model was solved analytically by M. Gorenstein, V. Petrov, G. Zinovjev, PLB 106 (1981)**

Working Tree of Exactly Solvable Statistical Models of Cluster Type: SMM I

In 2000 I solved the Statistical Multifragmentation Model (**SMM**) of nuclear liquid-gas phase transition (PT) using approach of solving Gas of Bags
No Coulomb interaction + thermodynamic limit

K. Bugaev, M. Gorenstein, I. Mishustin and W. Greiner, PRC 62 (2000)

Due to absence of surface tension of bags => GBM is highly unrealistic!

- GBM employes eigen volume approximation (valid for high densities!)**
- => it is unrealistic at low densities where the excluded volume must be used!**
- => no way to include the ordinary hadrons!**
- => unrealistic phase diagram: no critical endpoint and no cross-over!**

Available volume ($V - \sum_k v_k n_k$)
 v_k is eigen volume

High densities!

Eigen volume approximation means that bags move inside some cells!
 It is good for high densities!

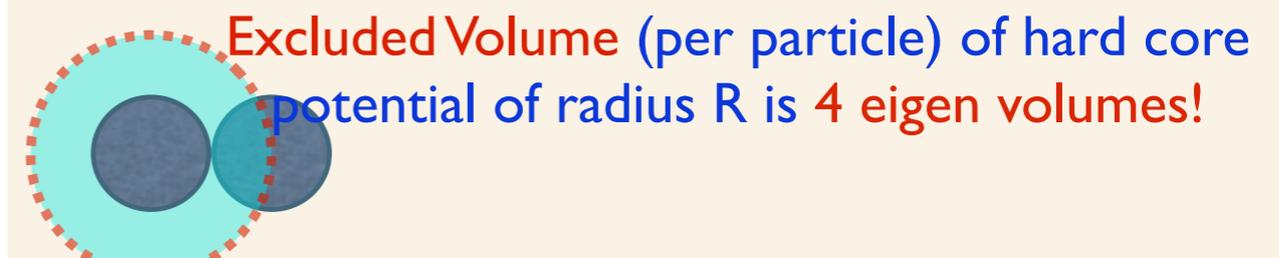


instead of $\left(V - \frac{\sum_{k,j} n_k b_{kj} n_j}{\sum_k n_k} \right)$

Low density approximation!

b_{kj} is excluded volume (second virial coeff)

Interaction: Hard core repulsion a la VDW



Working Tree ...: Scaling Relations of SMM (2001)

For simple liquids it was proven:

See M.E. Fisher, J. Math. Phys. 5 (1964) 944

$$\text{Fisher (1964) : } \alpha' + 2\beta + \gamma' \geq 2 ,$$

$$\text{Griffiths (1965) : } \alpha' + \beta(1 + \delta) \geq 2 ,$$

$$\text{Liberman (1966) : } \gamma' + \beta(1 - \delta) \geq 0 .$$

- For $\alpha' = 0 \implies$ new scaling relations:

For SMM
temperature
dependence of
surface tension

$$\alpha' + 2\beta + \gamma' = \frac{\zeta}{\sigma} = 15/8 < 2$$

$$\alpha' + \beta(1 + \delta) = \frac{\zeta}{\sigma} = 15/8 < 2$$

**P. T. Reuter and
K. A. Bugaev,
PLB 517 (2001)**

For Fisher model
T-dependence of
surface tension

$\zeta = 1; \sigma = \frac{2}{3}$ the SMM obeys stronger conditions!

We showed to M.E. Fisher that conditions of his famous proof are not fulfilled for the SMM and similar models. \implies This proof cannot be applied!

This was the first evidence that the standard definitions of crit. exponents are wrong!

Working Tree ...: Laplace-Fourier transform, SMM in finite volumes + Surface Entropy (2003-2006)

During my work at LBNL (2003-2006) I answered a few principal questions:

1. What is T-dependence of surface tension coefficient of nuclear matter?

It is Linear in T!

L. G. Moretto, K. A. Bugaev, J. B. Elliott, R. Ghetti, J. Helgesson and L. Phair,
Phys. Rev. Lett. (2005) 202701

Using Laplace-Fourier transform I **solved exactly**: Hills and Dale model for partition of surface deformations and SMM in finite volume
=> exact solutions showed me that

2. Above T_c the surface tension coefficient **can be negative**.

K. A. Bugaev, L. Phair, J. B. Elliott, **Phys. Rev. E** 72 (2005)

3. In finite systems the analog of mixed phase is **NOT** a superposition of two pure phases, it is gas + a set of metastable states **with complex free energy**

K. A. Bugaev, **Acta. Phys. Polon. B** 36 (2005)

Working Tree ...: Quark Gluon Bags with Surface Tension (2006-2010)

After my return to Kyiv in 2006, I included surface tension into Gas of Bags
Allowing **negative values of surface tension coefficient**
to exist above T_c

K. A. Bugaev, Phys. Rev. C 76 (2007)

After reading this work **G.M. Zinovjev said a great thing to me:«Lyakseich, if you are right, then we should see this conclusion in lattice QCD!»**

This was a turning point and in 2010 we showed (theorem!) that

When color tube breaks down (melting of hadrons and bags) =>

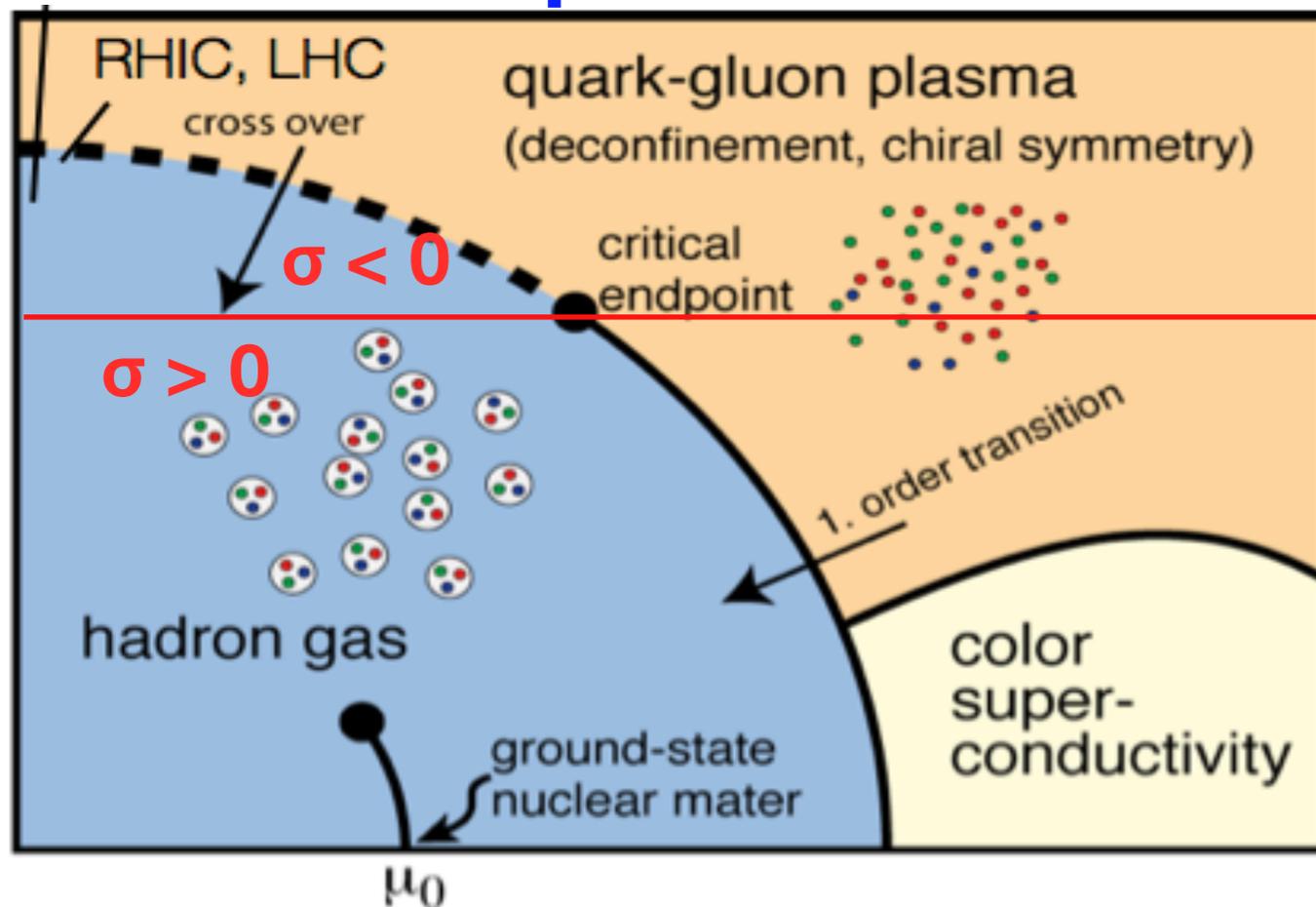
SURFACE TENSION COEFFICIENT of colour string is NEGATIVE!

K. A. Bugaev and G. M. Zinovjev, Nucl. Phys. A 848 (2010)

Role of Surface Tension above T_{cep}

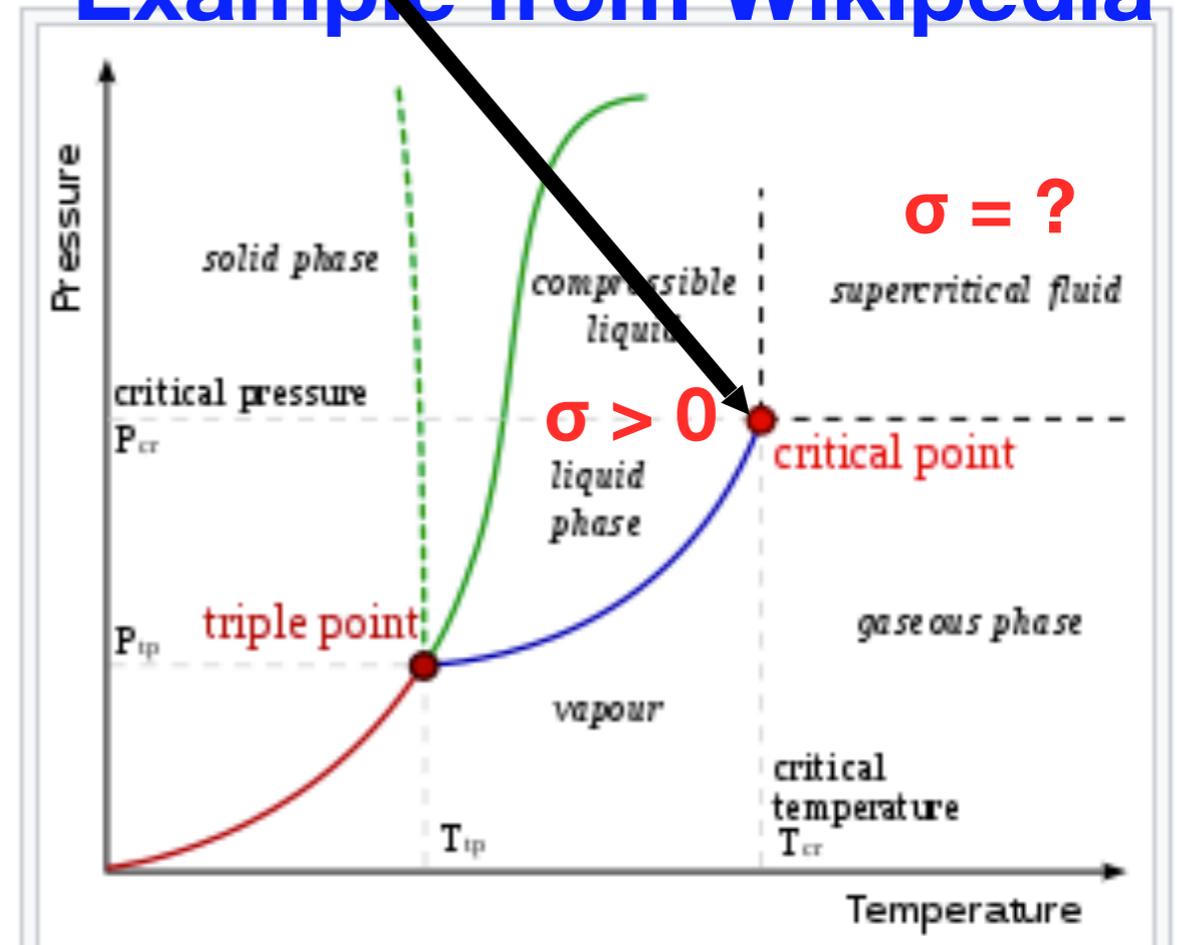
What is the physical reason that the 1-st Order PT curve is terminated? Experiments show that at (3)CEP the surface tension coefficient σ is 0, but what is σ at $T > T_{cep}$?

Our expectations



Baryonic chemical potential μ

Example from Wikipedia



A typical phase diagram for a single-component material, exhibiting solid, liquid and gaseous phases. The solid green line shows the usual shape of the liquid–solid phase line. The dotted green line shows the anomalous behavior of water when the pressure increases. The triple point and the critical point are shown as red dots.

So far, the only reason which may prevent the condensation of hadrons into a large bag is negative Surface tension coefficient for $T > T_{cep}$.

Working Tree ...: Role of Negative Surface Tension

K. A. Bugaev and G. M. Zinovjev, Nucl. Phys. A 848 (2010)

At T=0 the surface tension coefficient of quark gluon bags in SU3 color QCD is 147 MeV/fm².

A. G. Grunfeld^{1,2*} and G. Lugones^{3†}

Preprint 1804.09898v2 [nucl-th]

(2) Within the NJL model, surface tension in cold hybrid star conditions is ~ 150 MeV/fm². For such large values, mixed phases are mechanically unstable and the hadron-quark interface in a hybrid star should be a sharp discontinuity.

In 2012 we developed and solved the model of QGBags with Surface Tension with critical point and solved problem which arose in 1981

K. A. Bugaev, V. K. Petrov, G. M. Zinovjev, Phys. Part. Nucl. Lett. 9 (2012)

In 2012 we found the critical exponent of QGBags with Surface Tension with Critical point and found 1st non-Fisher universality class

A.I. Ivanytskyi, K.A. Bugaev, A.S. Sorin, G. M. Zinovjev, Phys. Rev. E 86 (2012)

Working Tree ...: Role of Finite Width in Colour Confinement (2009-2011)

Standard picture of color confinement: break of colour sting => leads to a pair of colorless fragments (hadrons or bags)

In 2009 we developed a paradox free statistical mechanics of QG bags with Finite width and found its parameters from lattice QCD data

In vacuum at T=0 the width of bag of mean mass $\langle M \rangle > 2.5$ GeV is

$$\Gamma = 450 \text{ MeV} [\langle M \rangle / 2 \text{ GeV}]^{0.5}$$

In medium at T= 160 MeV the width of bag of mean mass $\langle M \rangle > 2.5$ GeV is

$$\Gamma = 1400 \text{ MeV} [\langle M \rangle / 2 \text{ GeV}]^{0.5}$$

=> Even the QG bag is created one cannot detect it directly, but via decays it looks like a very wide resonance

K. A. Bugaev, V. K. Petrov, G. M. Zinovjev, Europhys. Lett. 85, (2009)

K. A. Bugaev, V. K. Petrov, G. M. Zinovjev, Phys. Rev. C 79, (2009)

=> at first glance the chance to ever detect QG bag was hopeless...

Working Tree ...: Apparent Width of Wide Resonances (2011-2015)

In 2015 we showed that in dense medium the QG bags acquire
Apparent width

$$\Gamma = T \ln(2)$$

and, hence, there is a hope to detect them via certain decays!

**K. A. Bugaev, A. I. Ivanytskyi, D. R. Oliinychenko, E. G. Nikonov, V. V. Sagun,
G. M. Zinovjev, Ukr. J. Phys. 60, (2015)**

Working Tree ...: Morphological Thermodynamics (2021-20...)

**Hope we will complete the Morphological Thermodynamics of
hadrons and QG bags and will discover something else!**

Many Happy Returns, dear Teacher!

Back up slides

Critical exponents of SMM (2001)

P. T. Reuter and K. A. Bugaev, PLB 517 (2001)

$\epsilon = (T_c - T) / T_c$ is relative deviation of temperature from T_c

k is number of nucleons in a fragment

	SMM	General case	Fisher Model
Surface tension	$\epsilon^{\frac{5}{4}} k^{\frac{2}{3}}$	$\epsilon^{\zeta} k^{\sigma}$	$\zeta = 1, \sigma = \frac{2}{3}$
α'	0	0	$2 - \frac{\zeta}{\sigma}(\tau_F - 1)$
β	$\frac{15}{8}(2 - \tau_F)$	$\frac{\zeta}{\sigma}(2 - \tau_F)$	$\frac{\zeta}{\sigma}(\tau_F - 2)$
γ'	$\frac{15}{4}(\tau_F - \frac{3}{2})$	$\frac{2\zeta}{\sigma}(\tau_F - \frac{3}{2})$	$\frac{\zeta}{\sigma}(3 - \tau_F)$
δ	$\frac{\tau_F - 1}{2 - \tau_F}$	$\frac{\tau_F - 1}{2 - \tau_F}$	$\frac{1}{\tau_F - 2}$

The FDM obeys all Scaling Relations exactly.

What are the Scaling Relations of the SMM?

Confinement by Color String before sQGP

Confinement = absence of free color charges

Consider confining string between static q & anti q of length L and radius $R \ll L$



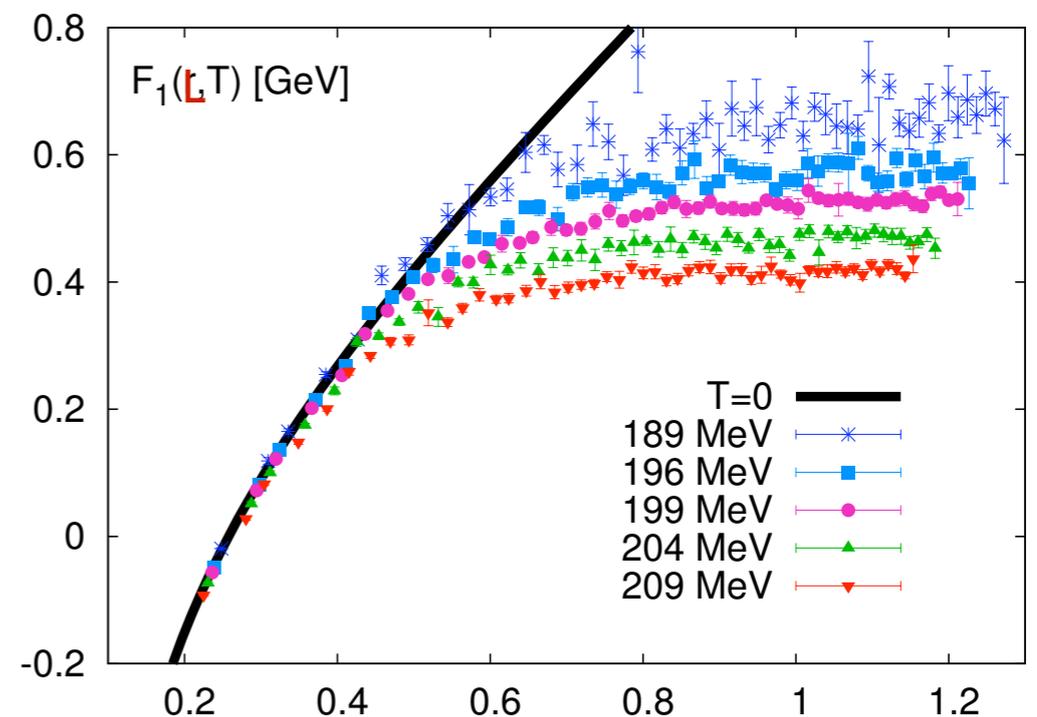
Its free energy measured from Polyakov loop correlator is $F_{str} = \sigma_{str} L$

Confinement means infinite free energy for infinite L

Deconfinement means that string tension vanishes

Can be rigorously found by Lattice QCD

At $T=0$ the string tension = 12 tons!



Coulomb part

confining part

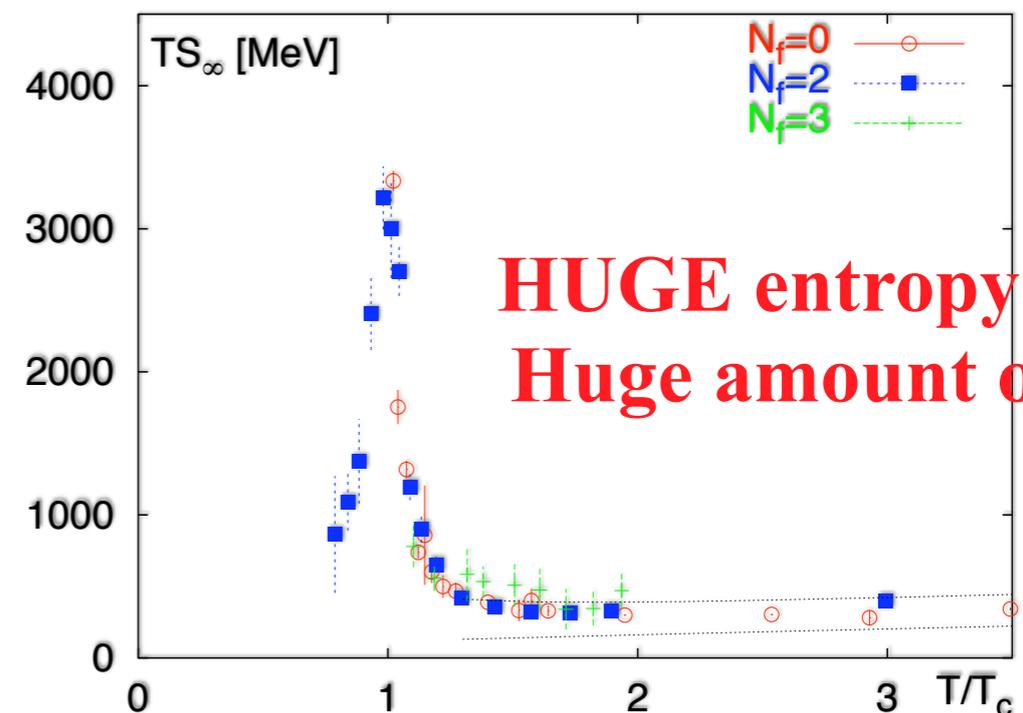
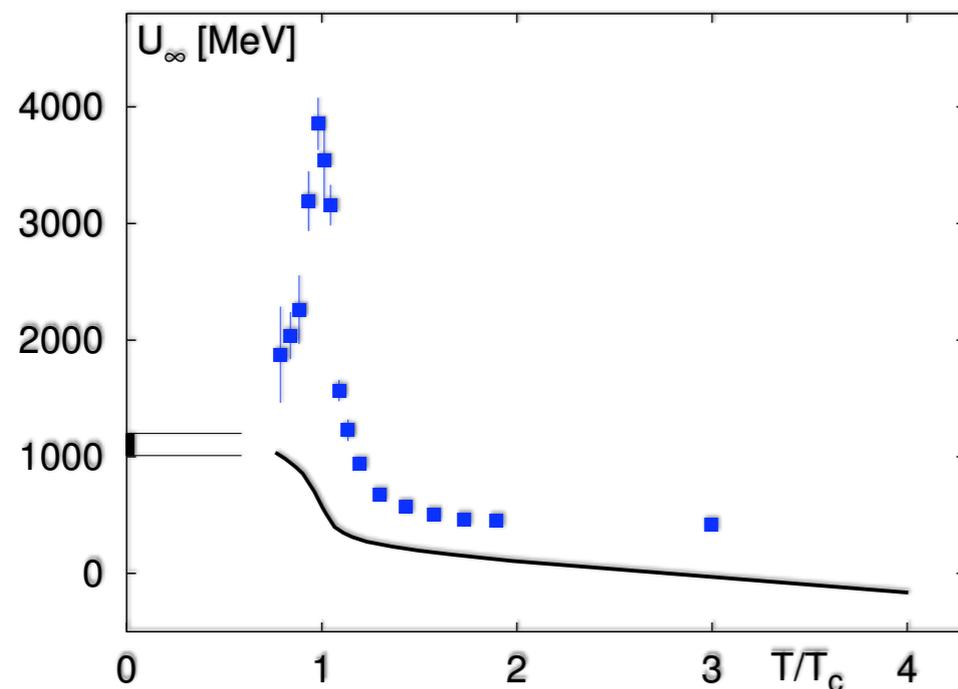
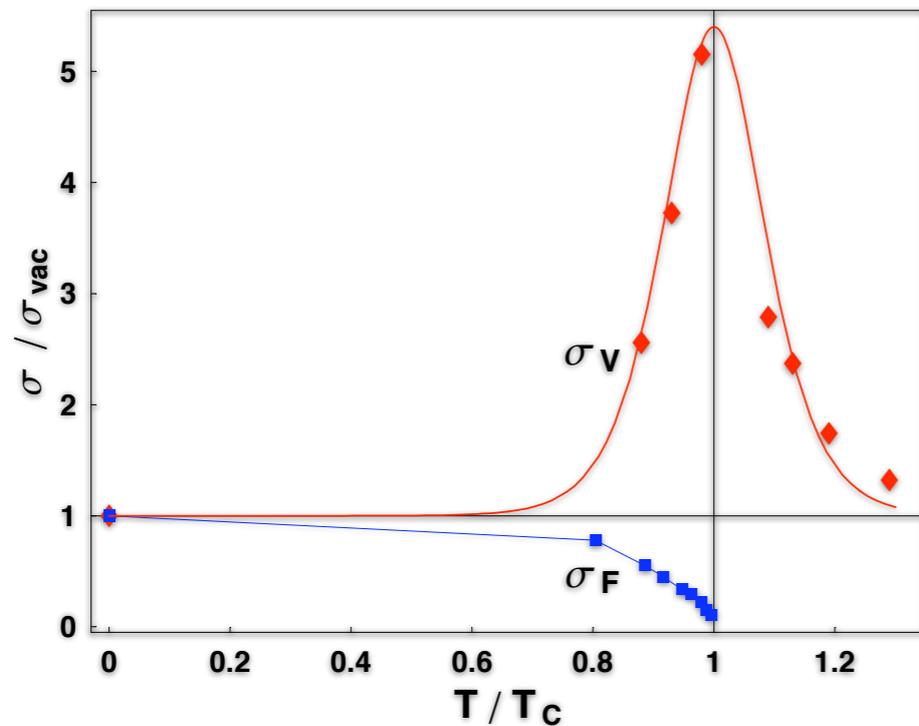
Confinement by Color String within sQGP

From $F \Rightarrow$ Internal energy U , entropy S

$$U(T, r) = F - TdF/dT = F + TS$$

String tension for internal energy (V) is finite

String tension σ_{str} for free energy (F) $\rightarrow 0$



HUGE entropy $S=20!$?
Huge amount of dof!?

Very strong interaction above $T_c!$ \Rightarrow No color charge separation!

String Tension vs Surface Tension

K.A.B., G.M. Zinovjev, Nucl. Phys. A848 (2010)

Identify now **colour string** as **cylindrical bag** of length L and radius $R \ll L$

Neglect effects of color sources and get cylinder FREE ENERGY as:

$$F_{cyl}(T, L, R) \equiv \underbrace{-p_v(T)\pi R^2 L}_{thermal} + \underbrace{\sigma_{surf}(T)2\pi RL}_{surface} + \underbrace{T\tau \ln \frac{V}{V_0}}_{small}$$

Equating the cylinder FREE ENERGY to string free energy $F_{str} = \sigma_{str}L$

$$\sigma_{str}(T) = \sigma_{surf}(T) 2\pi R - p_v(T)\pi R^2 + \cancel{\frac{T\tau}{L} \ln \left[\frac{\pi R^2 L}{V_0} \right]}$$

We got a new possibility to determine QGP bag surface tension directly from LQCD!

From bag model pressure $p_v(T = 0) = -(0.25)^4 \text{ GeV}^4$, $R = 0.5 \text{ fm}$ and $\sigma_{str}(T = 0) = (0.42)^2 \text{ GeV}^2 \Rightarrow$

$$\sigma_{surf}(T = 0) = (0.2229 \text{ GeV})^3 + 0.5 p_v R \approx \boxed{(0.183 \text{ GeV})^3} \approx 157.4 \text{ MeV fm}^{-2}.$$

Surface Tension at Cross-over

For vanishing σ_{str} one has $\sigma_{str}^{LQCD} \approx \frac{\ln(L/L_0)}{R^2} C$

This is due to increase of surface fluctuations \Rightarrow in general

$$\sigma_{str}(T) R^k \rightarrow \omega_k > 0 \quad \text{for} \quad k > 1$$

Parametrize $\sigma_{str} = \sigma_{str}^0 t^\nu$, where

$$t \equiv \frac{T_{tr}(\mu) - T}{T_{tr}(\mu)} \rightarrow +0$$

and find total pressure and total entropy density

for $\mu = 0$ (baryonic chemical potential)

$$p_{tot} = p_v(T) - \frac{\sigma_{surf}(T)}{R} \equiv \frac{\sigma_{surf}(T)}{R} - \frac{\sigma_{str}}{\pi R^2} \rightarrow \left[\frac{\sigma_{str}}{\omega_k} \right]^{\frac{1}{k}} \left[\sigma_{surf} - \frac{\omega_k}{\pi} \left[\frac{\sigma_{str}}{\omega_k} \right]^{\frac{k+1}{k}} \right]$$

$$s_{tot} = \left(\frac{\partial p_{tot}}{\partial T} \right)_\mu \rightarrow \underbrace{\frac{1}{k} \frac{\sigma_{str}}{\omega_k} \left[\frac{\sigma_{str}}{\omega_k} \right]^{\frac{1}{k}} \frac{\partial \sigma_{str}}{\partial T}}_{\text{dominant since } \sigma_{str} \rightarrow 0} \sigma_{surf} + \cancel{\left[\frac{\sigma_{str}}{\omega_k} \right]^{\frac{1}{k}} \frac{\partial \sigma_{surf}}{\partial T} - \frac{k+2}{\pi k} \left[\frac{\sigma_{str}}{\omega_k} \right]^{\frac{2}{k}} \frac{\partial \sigma_{str}}{\partial T}}$$

For finite σ_{surf} and $\frac{\partial \sigma_{str}}{\partial T} < 0$

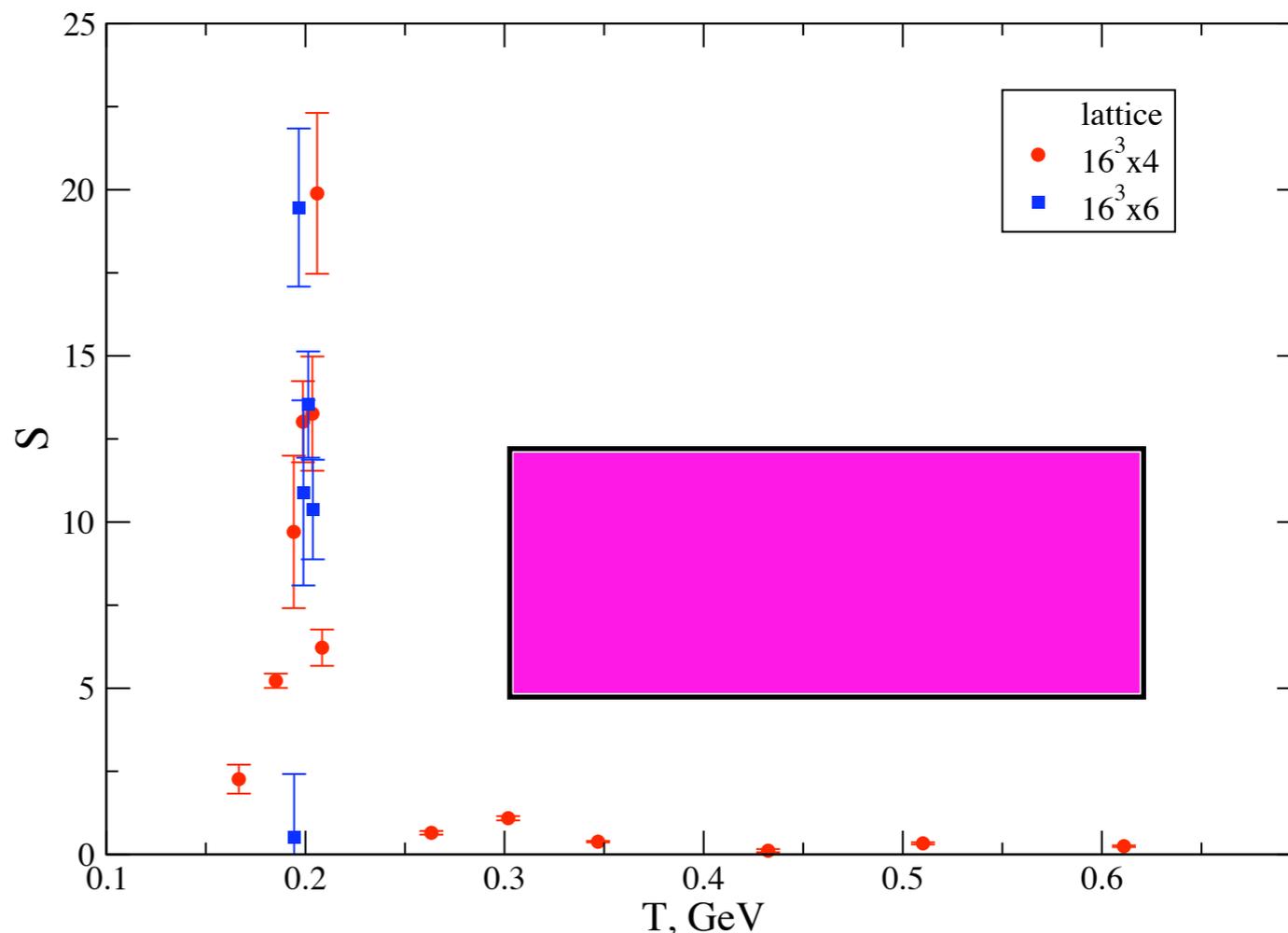
$\Rightarrow \sigma_{surf} < 0$ since $s_{tot} > 0$

Mysterious Maximum of Colour Tube Entropy

Low T no surface fluctuations
=> surface entropy is small



At high $T < T_c$ => very strong
surface fluctuations! =>
String entropy is large!
Huge number of dof!



Above T_c there is NO free surface =>
surface entropy = 0 !

This is a solution of Mysterious Maximum problem